

Energy Conversion

2nd year electrical

Chapter 3: Transformers

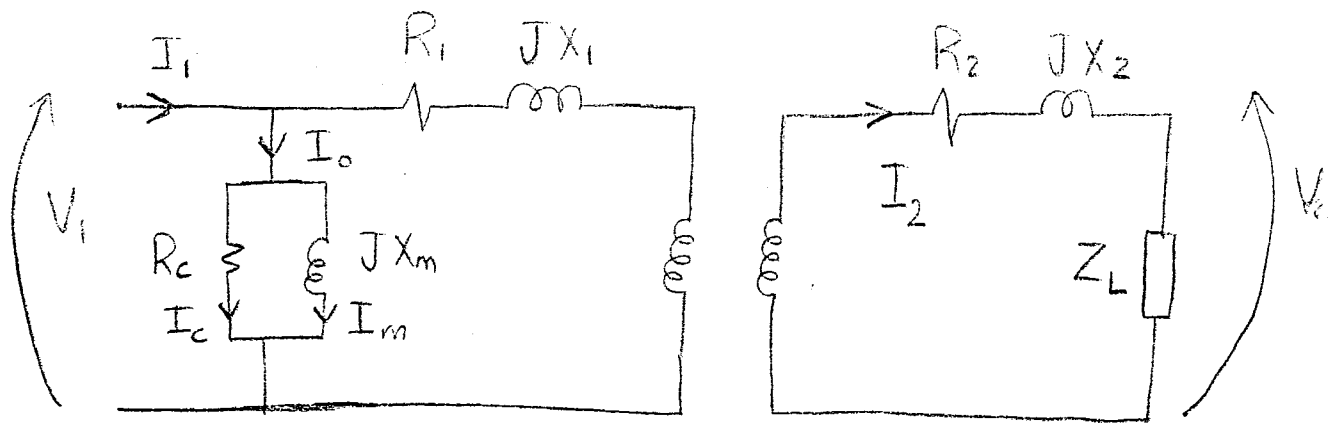
Part (2)

Paper: 3

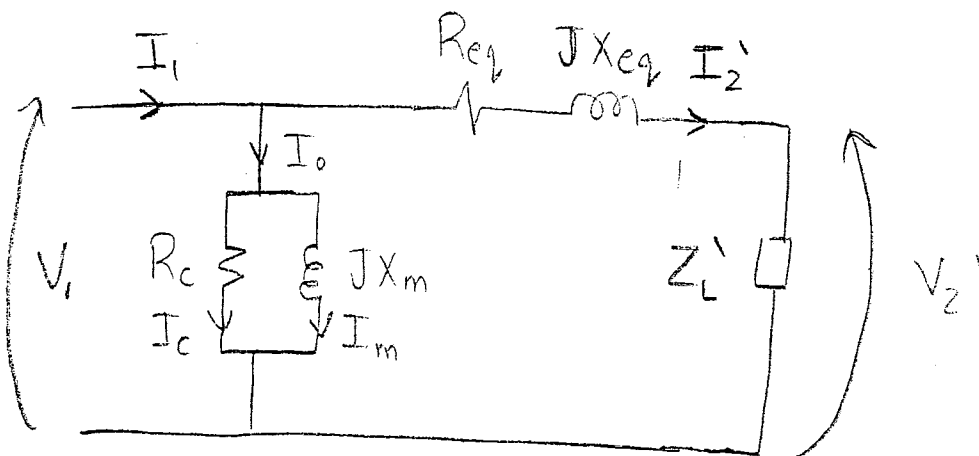
Transformers - Continued

- ① Approximate equivalent circuit
- ② Voltage regulation + phasor diagram
- ③ Efficiency (η)
- ④ Transformer tests $\begin{cases} \rightarrow \text{open circuit test} \\ \rightarrow \text{short circuit test} \end{cases}$
- ⑤ Separation of iron losses $\begin{cases} \rightarrow P_{\text{hysteresis}} \\ \rightarrow P_{\text{eddy}} \end{cases}$

① Approximate equivalent circuit



→ Referring to primary side



$$* R_{eq} = R_1 + R_2'$$

$$R_{eq} = R_1 + a^2 R_2$$

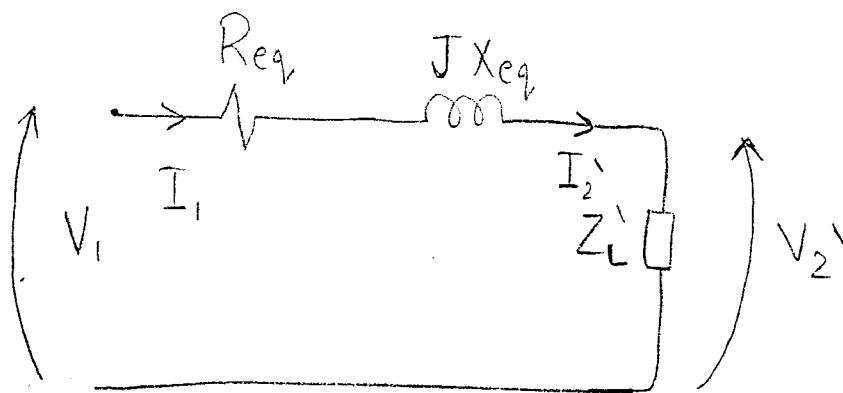
$$* X_{eq} = X_1 + X_2'$$

$$X_{eq} = X_1 + a^2 X_2$$

② Transformer Voltage regulation (VR)

Definition: It's a measure of the change of voltage from no load to any loading condition. ($VR \leq 5\%$)

→ Consider the equivalent circuit of the transformer as follows:



$$\text{Voltage regulation} = \frac{V_1 - V_2'}{V_1}$$

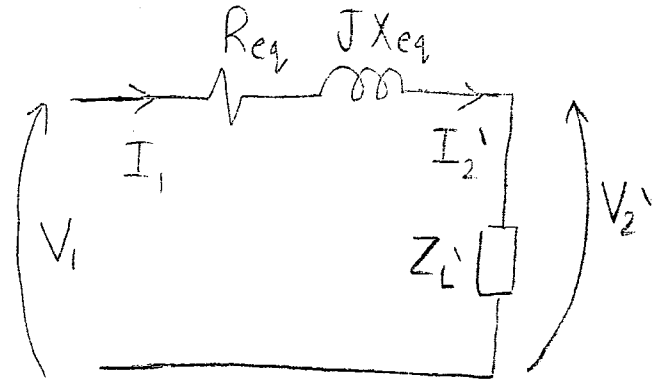
→ at no-load → $I_2' = 0$ → $V_1 = V_{2' \text{ no-load}}$

$$\therefore \text{Voltage regulation} = \frac{V_1 - V_2'}{V_1} = \frac{V_{2' \text{ no-load}} - V_2'}{V_{2' \text{ no-load}}}$$

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Voltage regulation For lagging p.f load

$$VR = \frac{V_1 - V_2'}{V_1}$$

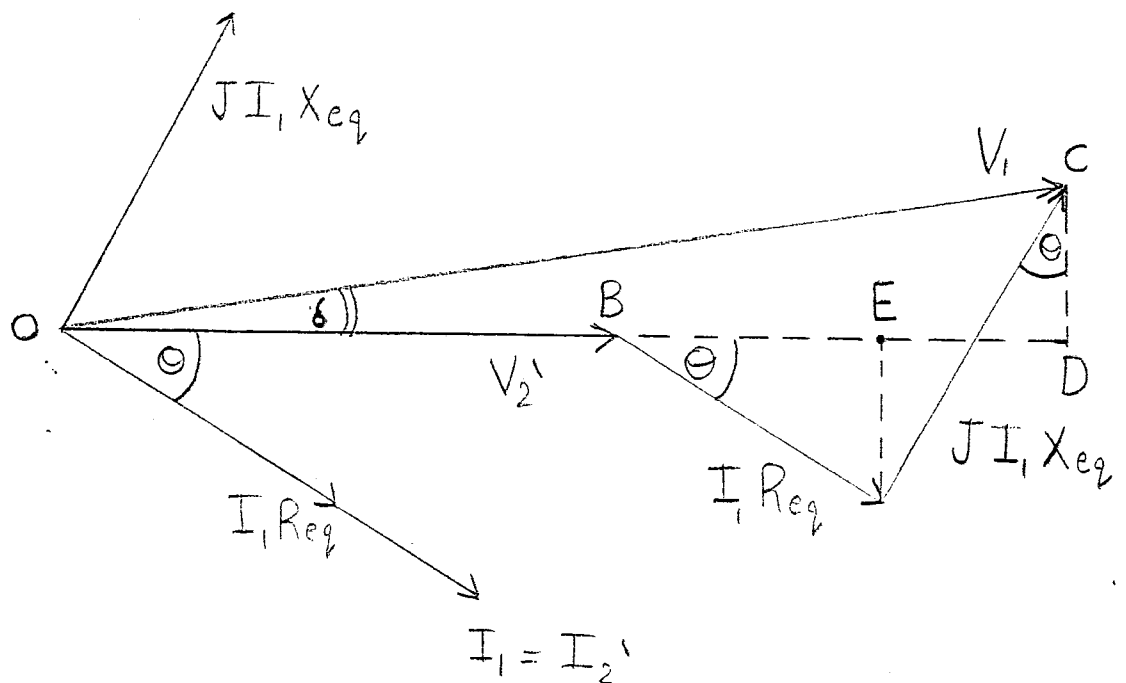


→ KVL

$$\overline{V_1} = \overline{V_2} + \overline{I_1} (R_{eq} + jX_{eq})$$

$$\overline{V_1} = \overline{V_2} + \overline{I_1} R_{eq} + j\overline{I_1} X_{eq} \rightarrow \text{phasor equation}$$

→ phasor diagram



To get $|V_1|$

$$|V_1| = OC \approx OD \text{ (as } \delta \text{ is very small angle)}$$

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$$|V_1| = OD = OB + BE + ED$$

$$= |V_2'| + I_1 R_{eq} \cos \theta + I_1 X_{eq} \sin \theta$$

$$\therefore |V_1| = |V_2'| + I_1 R_{eq} \cos \theta + I_1 X_{eq} \sin \theta$$

$$|V_1| - |V_2'| = I_1 R_{eq} \cos \theta + I_1 X_{eq} \sin \theta$$

$$\therefore VR = \frac{|V_1| - |V_2'|}{|V_1|}$$

$$\therefore VR = \frac{I_1 R_{eq} \cos \theta + I_1 X_{eq} \sin \theta}{V_1} \rightarrow \text{For lagging PF}$$

→ Condition for Maximum Voltage regulation

$$\frac{d(VR)}{d\theta} = 0 \Rightarrow \frac{d}{d\theta} \left[\frac{I_1 R_{eq} \cos \theta + I_1 X_{eq} \sin \theta}{V_1} \right] = 0$$

$$\therefore I_1 R_{eq} (-\sin \theta) + I_1 X_{eq} \cos \theta = 0$$

$$\therefore \tan \theta = \frac{X_{eq}}{R_{eq}}$$

∴ The load angle for maximum VR is

$$\theta_{\max} = \tan^{-1} \frac{X_{eq}}{R_{eq}} \quad (\text{lagging pf})$$

$$\therefore VR_{\max} = \frac{I_1 R_{eq} \cos \theta_{\max} + I_1 X_{eq} \sin \theta_{\max}}{V_1}$$

③ Efficiency of transformers (η)

$$\eta = \frac{P_{o/p}}{P_{i/p}} = \frac{P_{o/p}}{P_{o/p} + P_{loss}}$$

where $P_{loss} = P_{cu} + P_{core}$

\Rightarrow To get a general expression for $\eta \rightarrow$ فالم

$$\eta = \frac{P_{o/p}}{P_{o/p} + P_{cu} + P_{core}}$$

* Let $X = \text{loading ratio} = \text{loading factor}$

$$X = \frac{I_2}{I_{2f.L}}$$

$$X = \frac{I_2 * V_2}{I_{2f.L} * V_2} = \frac{S}{S_{f.L}}$$

$$\therefore X = \frac{I_2}{I_{2f.L}} = \frac{S}{S_{f.L}}$$

$$\Rightarrow \textcircled{1} P_{o/p} = V_2 I_2 \cos \phi_L = V_2 I_{2f.L} * \left(\frac{I_2}{I_{2f.L}} \right) \cos \phi_L$$

$$P_{o/p} = X S_{f.L} \cos \phi_L \rightarrow \textcircled{1}$$

$$\Rightarrow \textcircled{2} P_{core} = \text{Constant} \rightarrow \textcircled{2}$$

$$\Rightarrow \textcircled{3} P_{cu} \propto I^2$$

$$P_{cu} = X^2 P_{cu f.L} \rightarrow \textcircled{3}$$

From $\textcircled{1}, \textcircled{2}, \textcircled{3}$ We get:

$$\eta = \frac{X S_{f.L} \cos \phi_L}{X S_{f.L} \cos \phi_L + X^2 P_{cu f.L} + P_{core}} \quad \text{---} \rightarrow$$

To get η_{max}

$$\frac{d\eta}{dX} = 0 \Rightarrow X \Big|_{\eta_{max}} = \sqrt{\frac{P_{core}}{P_{cu f.L}}}$$

$$\therefore \eta_{max} = \frac{\left(\sqrt{\frac{P_{core}}{P_{cu f.L}}} \right) * S_{f.L} * \cos \phi_L}{\left(\sqrt{\frac{P_{core}}{P_{cu f.L}}} \right) * S_{f.L} * \cos \phi_L + 2 P_{core}} \quad \text{---} \rightarrow$$

Note: Transformer efficiency is Very high ($\eta_{trans} \approx 95\%$) as it has no rotational losses

Ex: ①

The parameters of the equivalent circuit of a 10KVA, 50HZ, 2300/230V distribution transformer are:

$$r_1 = 3.6 \Omega, X_1 = 15.8 \Omega$$

$$r_2 = 0.0396 \Omega, X_2 = 0.158 \Omega$$

Where 1, 2 refers to HV, LV respectively

- ① IF the transformer delivers its rated KVA at 0.8 pf lagging to a load on low-tension (LV) side, calculate Voltage regulation.
- ② Calculate the efficiency in part (a) if the iron losses are 75W at rated Voltage.

Solution

(a) To get I_2 rated at LV side

$$|I_{2 \text{ rated}}| = \frac{S_{\text{rated}}}{V_2} = \frac{10\,000}{230} = 43.48 \text{ A}$$

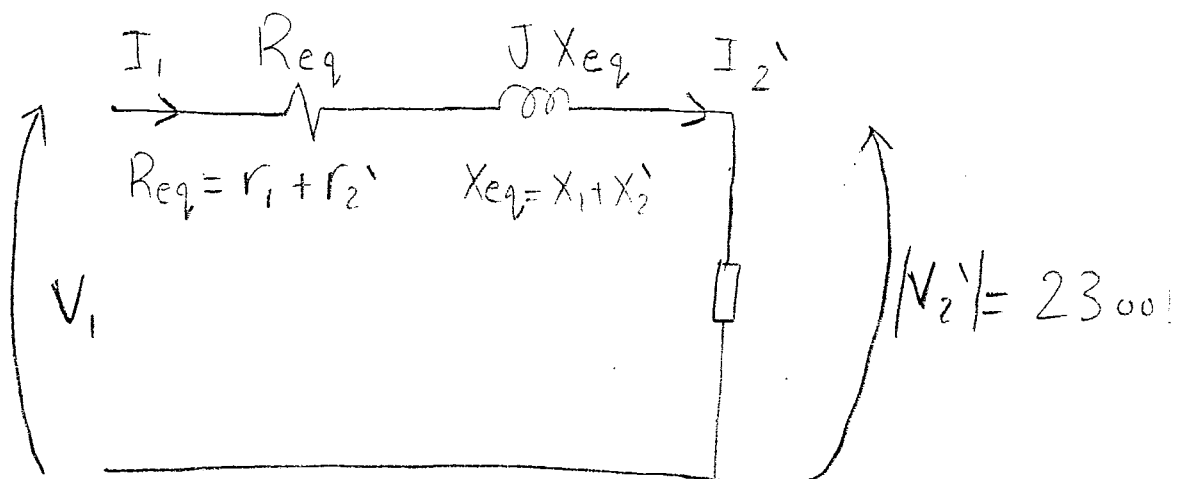
Referring to HV side

$$|V_2'| = a |V_2| = 10 \times 230 = 2300 \text{ V}$$

$$|I_2'| = \frac{1}{a} |I_2| = \frac{1}{10} \times 43.48 = 4.348 \text{ A}$$

$$r_2' = a^2 r_2 = (10)^2 \times 0.0396 = 3.96 \Omega$$

$$X_2' = a^2 X_2 = (10)^2 \times 0.158 = 15.8 \Omega$$



$$R_{eq} = r_1 + r_2' = 7.56 \Omega$$

$$X_{eq} = X_1 + X_2' = 31.6 \Omega$$

$$\phi = \cos^{-1} 0.8 = 36.87^\circ \text{ (lagging)}$$

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using laws of Voltage regulation: ($I_1 = I_2'$)

$$V_1 - V_2' = I_1 R_{eq} \cos \phi + I_1 X_{eq} \sin \phi$$

$$V_1 = V_2' + I_2' R_{eq} \cos \phi + I_2' X_{eq} \sin \phi$$

$$\therefore V_1 = 2300 + (4.348)(7.56) \cos(36.87) \\ + (4.348)(31.6) \sin(36.87)$$

$$V_1 = 2408.7 \text{ V}$$

$$\therefore VR = \frac{V_1 - V_2'}{V_1} = \frac{2408.7 - 2300}{2408.7}$$

$$VR = 4.5\%$$

(b) Efficiency

$$\eta = \frac{X S_{f.L} \cos \phi}{X S_{f.L} \cos \phi + X^2 P_{cu.f.L} + P_{core}}$$

* $X = 1$

* $S_{f.L} = 10000 \text{ VA}$, $\cos \phi = 0.8$

* $P_{cu.f.L} = (I_2')^2 R_{eq} = (4.348)^2 * 7.56 = \underline{143 \text{ W}}$

* $P_{core} = P_{iron} = \underline{75 \text{ W}}$

$$\therefore \eta = 97.34\%$$

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pb (7 ii) Sheet 3

Given: • $S_{F.L} = 500 \text{ kVA}$

$$\bullet \eta|_{x=1} (\text{Full load}) = 0.95$$

$$\bullet \eta|_{x=0.6} = 0.95$$

$$\bullet \text{p.f} = 1$$

(a) separate out the transformer losses

using the law of η

$$\eta = \frac{P_o/p}{P_i/p} = \frac{P_o/p}{P_o/p + P_{\text{loss}}} = \frac{P_o/p}{P_o/p + P_{\text{Cu}} + P_{\text{core}}}$$

$$\eta = \frac{x S_{F.L} \cos \phi_L}{x S_{F.L} \cos \phi_L + x^2 P_{\text{Cu}_{F.L}} + P_{\text{core}}}$$

$$\eta|_{x=1} = \eta|_{x=0.6} = 0.95$$

$$\Rightarrow \eta|_{x=1} = 0.95$$

$$\frac{1 \times 500 \times 1}{1 \times 500 \times 1 + (1)^2 P_{Cu_{f.L}} + P_{core}} = 0.95$$

$$\therefore P_{Cu_{f.L}} + P_{core} = 26.31 \text{ KW} \rightarrow \textcircled{1}$$

$$\Rightarrow \eta|_{x=0.6} = 0.95$$

$$\frac{0.6 \times 500 \times 1}{0.6 \times 500 \times 1 + (0.6)^2 P_{Cu_{f.L}} + P_{core}} = 0.95$$

$$\therefore 0.36 P_{Cu_{f.L}} + P_{core} = 15.79 \text{ KW} \rightarrow \textcircled{2}$$

Solving ① & ② \Rightarrow

$$P_{Cu_{f.L}} = 16.4 \text{ KW}$$

$$P_{core} = 9.87 \text{ KW}$$

$$\textcircled{b} \quad \eta|_{x=0.75}, p.f = 1$$

$$\eta|_{x=0.75} = \frac{0.75 \times 500 \times 1}{0.75 \times 500 \times 1 + (0.75)^2 \times 16.4 + 9.87}$$

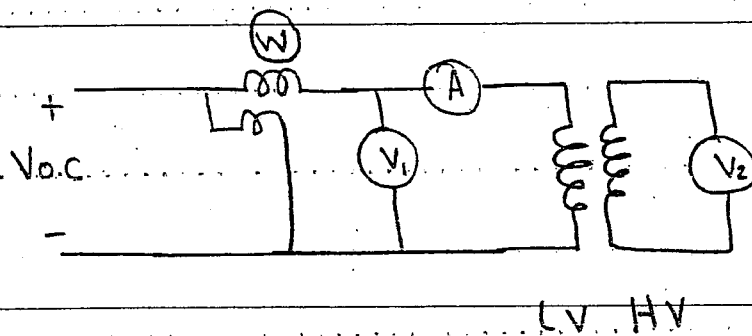
$$\therefore \eta|_{x=0.75} = 95.15 \%$$

#

(IV) Transformer Tests:

(1) Open circuit Test:

• This test is performed to find (R_c & X_m)



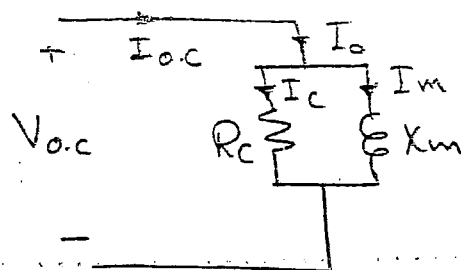
• $V_{o.c} \equiv$ Applied rated voltage $\equiv V_1$

• $I_{o.c.} \equiv$ open circuit current $\equiv A$

• $P_{o.c.} \equiv$ open circuit transformer power $\equiv W$

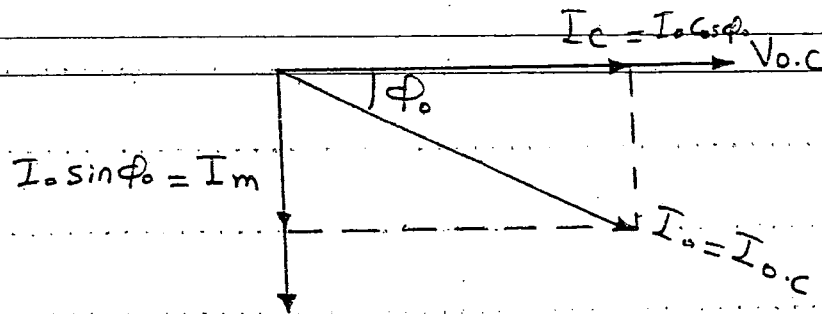
• How to get R_c & X_m ?

• The equivalent circuit of the transformer during the open circuit test will be:



$$\therefore R_c = \frac{V_o \cdot c}{I_c} \quad , \quad X_m = \frac{V_o \cdot c}{I_m}$$

∴ The values of I_c & I_m can be calculated through:



∴ $P_o \cdot c$ will be used to get ϕ_o , then using the above phasor we draw $I_o \cdot c$ lagging $V_o \cdot c$ by ϕ_o , then resolve $I_o \cdot c$ to (I_c & I_m).

∴ $P_o \cdot c = V_o \cdot c \cdot I_o \cdot c \cdot \cos \phi_o \Rightarrow \phi_o$ can be calculated.

$$\therefore \cos \phi_o = \frac{P_o \cdot c}{(V_o \cdot c)(I_o \cdot c)} \Rightarrow \phi_o = o.k.$$

$$\therefore I_c = I_o \cos \phi_o \text{ \& } I_m = I_o \sin \phi_o$$

$$\therefore R_c = \frac{V_o \cdot c}{I_o \cdot c} \text{ \& } X_m = \frac{V_o \cdot c}{I_m}$$

N.B :

$$[1] \quad I_m = \sqrt{I_{o.c}^2 - I_c^2} \quad (\text{Another way})$$

[2] R_c & X_m are referred to (L.V) side

[3] $I_{o.c}$ = no-load current

[4] $\cos \phi_0$ = no-load power factor

[5] $V_{o.c} \equiv$ rated voltage of the transformer if not given

[6] Instruments are connected to L.V side (why?)

(نظر - نظری)

... usually ($I_{o.c} = I_0$) is very small compared to the full load current ($\approx 0.5 I_{full\ load}$)

... This small current is measured through the L.V. of the transformer for better accuracy.

N.B

... usually the current in H.V side is lower than that of L.V

$$\therefore V_1 I_1 = I_2 V_2 \Rightarrow \frac{I_2}{I_1} = \frac{V_1}{V_2}$$

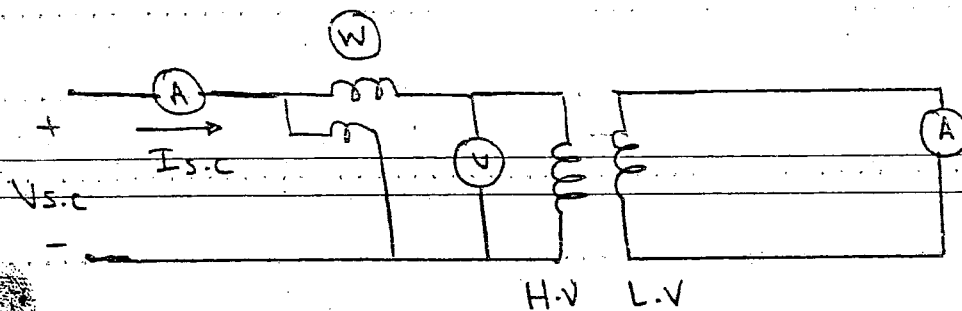
$$\rightarrow \text{If } \frac{V_1}{V_2} > 1$$

$$\therefore I_2 > I_1 \Rightarrow \text{our case.}$$

(15)

(2) Short Circuit test:-

- It is performed to find R_{eq} & X_{eq} .

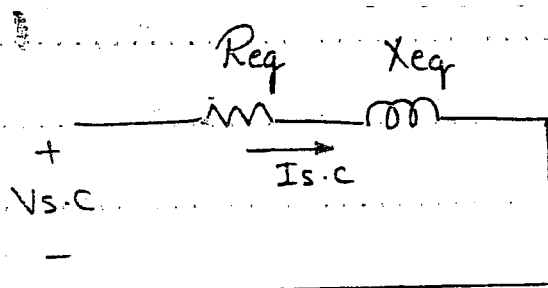


- $V_{s.c} = 5-10\%$ of rated voltage of H.V side $\equiv (V)$

- $I_{s.c} =$ short circuit transformer current $\equiv (A)$

- $P_{s.c} =$ short circuit transformer power $\equiv (W)$

- The equivalent circuit during this test:



- The $(R_c \& X_m)$ branch is neglected as the current (I_0) is very small with respect to $I_{s.c}$.

- To get (R_{eq}) :

$$R_{eq} = \frac{P_{s.c}}{I_{s.c}^2}$$

• To get X_{eq} :

$$Z_{eq} = \sqrt{R_{eq}^2 + X_{eq}^2} = \frac{V_{s-c}}{I_{s-c}}$$

$$\therefore X_{eq} = \sqrt{\left(\frac{V_{s-c}}{I_{s-c}}\right)^2 - R_{eq}^2}$$

N.B:

- ① $I_{s-c} \approx I_{f.L}$ of transformer H.V. side.
- ② $(R_{eq} \& X_{eq})$ are referred to the H.V. side.
- ③ The instruments are placed on H.V. side. (why?)
(وہاں پر اے. مپ. ہے)
In this test the Current flowing (I_{s-c}) is very high ($\approx I_{f.L}$).
This Current will be very high in the L.V. side and it will very difficult to measure it by an ammeter.
The voltage ($V_{s-c} \approx 5-10\% V_{rated}$), so it will be more accurate to measure it using the H.V. side.

[4] you can use:

$$r_1 = r_2' = \frac{R_{eq}}{2}$$

&

$$x_1 = x_2' = \frac{X_{eq}}{2}$$

[5] The open circuit test & short circuit test may be performed on any side, but it will be mentioned in the problem.

[6] Default: (not mentioned in the problem).

• open circuit test \Rightarrow on L.V. side

• short circuit test \Rightarrow on H.V. side.

[7] Cross-section Area of LV winding is large as it carries higher current
 $\therefore R_{LV}$ is small ($R = \frac{\rho L}{A}$)

[8] Cross-section Area of HV winding is small
 $\therefore R_{HV}$ is high. (18)

Ex (2)

A 50 KVA, 2200/220 V transformer

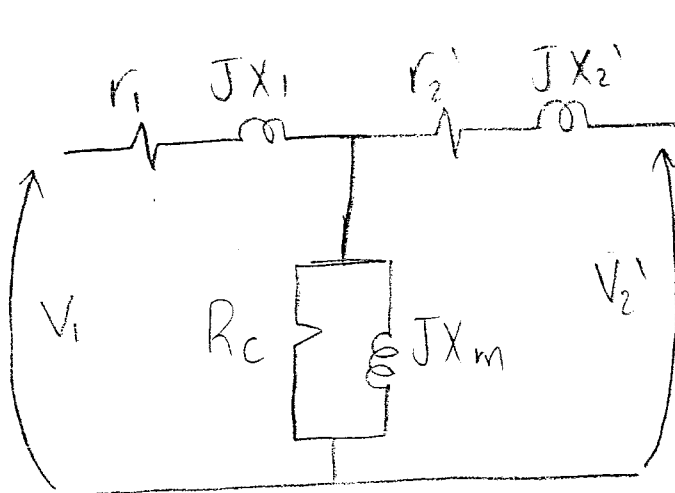
When tested give the following results:

open circuit test: Measurements on LV side
40 SW, 5A, 220V

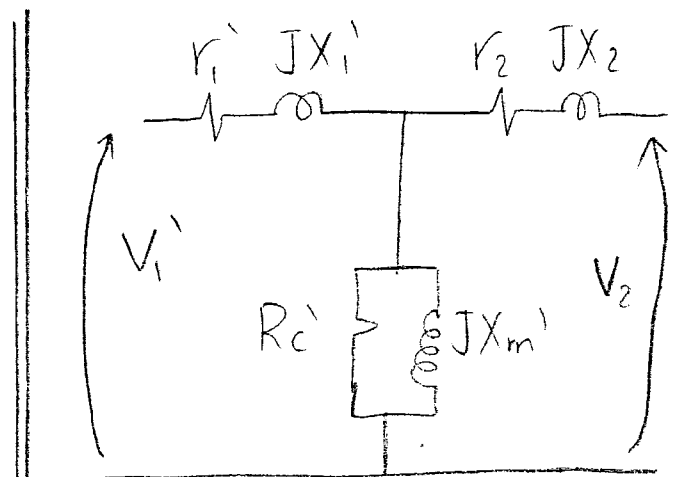
Short circuit test: Measurements on HV side
80 SW, 20.2 A, 95V

Find equivalent circuit parameters in ohms referred to HV side & LV side

Solution HV side \rightarrow primary side in this problem



Exact equivalent circuit
Referred to HV side



Exact equivalent circuit
Referred to LV side

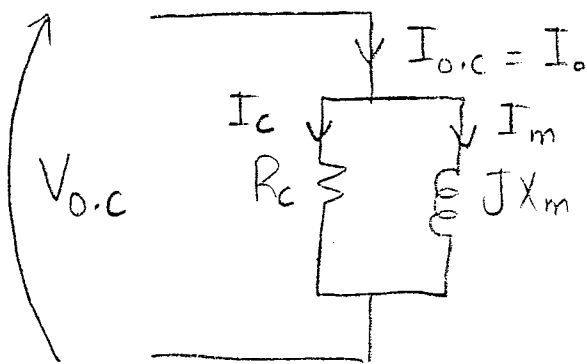
Solution

open circuit test

$$P_{o.c} = 40 \text{ SW}$$

$$I_{o.c} = 5 \text{ A}$$

$$V_{o.c} = 220 \text{ V}$$



$$\rightarrow * P_{o.c} = V_{o.c} I_{o.c} \cos \phi$$

$$\cos \phi = \frac{40 \text{ S}}{220 \times 5} = 0.368$$

$$\boxed{\phi = 68.4^\circ}$$

$$\rightarrow I_c = I_{o.c} \cos \phi = 1.84$$

$$I_m = I_{o.c} \sin \phi = 4.65$$

$$\rightarrow R_c|_{LV} = \frac{V_{o.c}}{I_c} = 119.56 \Omega$$

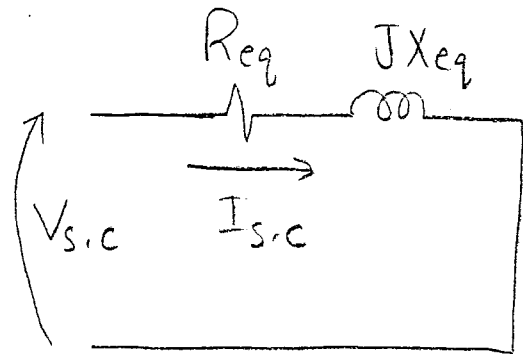
$$X_m|_{LV} = \frac{V_{o.c}}{I_m} = 47.3 \Omega$$

Short circuit test

$$P_{s.c} = 80 \text{ SW}$$

$$I_{s.c} = 20.2 \text{ A}$$

$$V_{s.c} = 95 \text{ V}$$



$$\rightarrow R_{eq} = \frac{P}{I_{s.c}^2}$$

$$R_{eq}|_{HV} = \frac{80 \text{ S}}{(20.2)^2} = 1.97 \Omega$$

$$\rightarrow Z_{eq} = \frac{V_{s.c}}{I_{s.c}} = 4.7$$

$$\rightarrow X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$

$$X_{eq}|_{HV} = 4.26 \Omega$$

$$\therefore R_{eq}|_{HV} = 1.97 \Omega$$

$$X_{eq}|_{HV} = 4.26 \Omega$$

(20)

Exact equivalent circuit referred to HV side

$$* R_{c|HV} = R_{c|LV} \left(\frac{N_{HV}}{N_{LV}} \right)^2 = 119.56 \left(\frac{2200}{220} \right)^2 = 11.95 \text{ k}\Omega$$

$$* X_m|_{HV} = X_m|_{LV} \left(\frac{N_{HV}}{N_{LV}} \right)^2 = 47.3 \left(\frac{2200}{220} \right)^2 = 4.73 \text{ k}\Omega$$

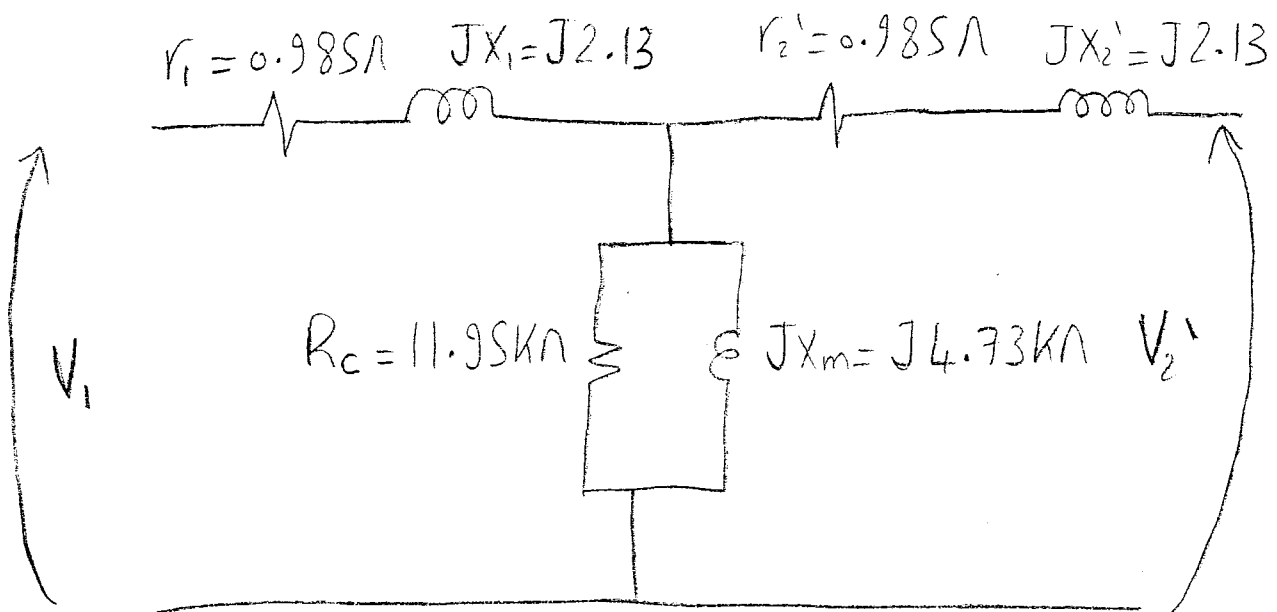
$$* R_{eq|HV} = 1.97$$

$$R_{eq|HV} = r_1 + r_2' \quad (r_1 \approx r_2')$$

$$\therefore r_1 = r_2' = \frac{R_{eq|HV}}{2} = 0.985 \Omega$$

$$* X_{eq|HV} = 4.26$$

$$\therefore X_1 = X_2' = \frac{4.26}{2} = 2.13$$

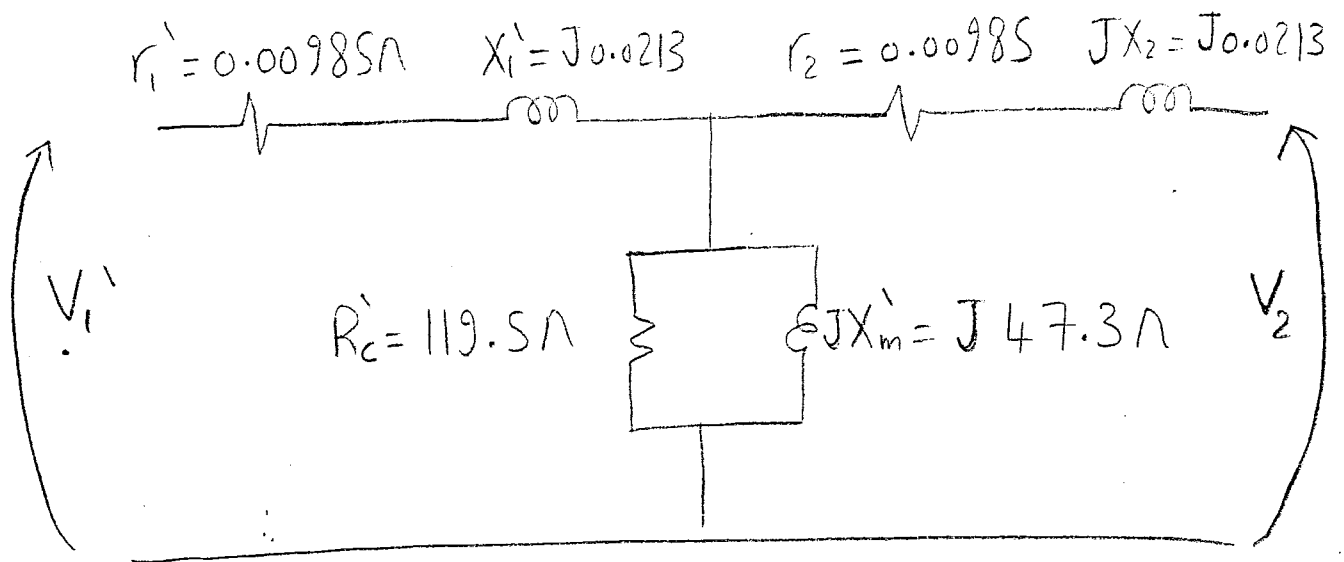


Exact equivalent circuit referred to
LV side

Referring the circuit of HV side
to LV side

$$r|_{LV} = r|_{HV} \left(\frac{N_{LV}}{N_{HV}} \right)^2 = r|_{HV} \left(\frac{220}{2200} \right)^2$$

$$r|_{LV} = \frac{r|_{HV}}{100}$$



Separation of iron losses

$$\rightarrow P_{\text{iron}} = P_{\text{core}} = P_h + P_e$$

\rightarrow where P_h : hysteresis losses

P_e : eddy losses

$$* P_h = K_h f (B_{\text{max}})^x$$

X: Steinmetz Constant (X=1.6)

$$(P_h = K_h f (B_{\text{max}})^{1.6})$$

دیا

$$* P_e = K_e f^2 (B_{\text{max}})^2$$

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$$\text{Where } B_{\text{max}} = \frac{V}{4.44 f N A}$$

$$\therefore P_h = K_h f \left(\frac{V}{4.44 f N A} \right)^x$$

$$P_h = K_h' V^x f^{1-x}$$

$$P_e = K_e f^2 \left(\frac{V}{4.44 f N A} \right)^2$$

$$P_e = K_e' V^2$$

Ex (3):

In a transformer, the Core loss is found to be 52 W at 40 Hz and 90 W at 60 Hz measured at the same peak flux density. Compute the hysteresis and eddy current losses at 50 Hz

Solution

$$P_h = K_h F (B_{\max})^{1.6}$$

$$\therefore B_{\max} = \text{Constant}$$

$$P_h = K_h' F$$

$$P_e = K_e F^2 B_{\max}^2$$

$$\therefore B_{\max} = \text{Constant}$$

$$\therefore P_e = K_e' F^2$$

$$\therefore P_{\text{core}} = P_h + P_e$$

$$P_{\text{core}} = K_h' F + K_e' F^2$$

using the given conditions

$$P_{\text{core}} = 52 \text{ W at } 40 \text{ Hz} \Rightarrow 52 = K_h' (40) + K_e' (40)^2 \rightarrow ($$

$$P_{\text{core}} = 90 \text{ W at } 60 \text{ Hz} \Rightarrow 90 = K_h' (60) + K_e' (60)^2 \rightarrow ($$

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Solving ①, ② $\Rightarrow K_h' = 0.9$ $K_e' = 0.01$

\therefore at 50Hz

$$P_{core} = K_h'(S_o) + K_e'(S_o)^2$$

$$P_{core} = 0.9(S_o) + 0.01(S_o)^2$$

$$P_{core} = 4S + 2S$$

$$\therefore P_{core} = 70W$$

$$P_h = 4SW$$

$$P_e = 2SW$$